## NOTATION

$D_{B}$, mean bubble diameter across the bed; $D$, column diameter; $k, k_{1}, k_{2}$, dimensionless coefficients; g, gravitational acceleration; $u$, $u_{0}$, velocity of filtration and velocity at initiation of fluidization; $H, H_{0}$, height of bed and height of motionless bed; h, vertical height above gas-distribution grid; $p=H / H_{0}$, expansion of bed; $F r=\left(u-u_{0}\right)^{2} / \mathrm{gH}_{0}$, Froude number; $u_{B}, v_{B}, \bar{v}_{B}$, absolute rate of bubble rise, relative rate of bubble rise, and relative rate of bubble rise averaged across the bed; $\tilde{u}_{B}$, $\tilde{v}_{B}$, local values of bubble velocities; $d$, diameter of solid particles; $\varepsilon_{0}$, bed porosity at initiation of fluidization.

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## CALCULATION OF CHARACTERISTICS OF A DIRECT-CURRENT

ARGON ARC
A. S. Sergienko and G. A. Fokov

UDC 533.9.082.15

The radial temperature profile of a cylindrical argon arc is calculated by method based on an elliptical approximation of the function $\sigma(S)$.

Article [1] presented the results of a study of static volt-ampere characteristics of a dc argon arc burning in a cylindrical channel formed by a set of water-cooled electrically neutral copper sections with inner diameter of $d=6 \mathrm{~mm}$. The current varied from 30 to 110 A at an argon flow rate of $G=0.03 \mathrm{~g} / \mathrm{sec}$.

For various practical applications it is important to theoretically calculate the temperature distribution over the section of the arc conductive channel. If we consider that the electrical energy introduced into a unit volume of cylindrically symmetric positive arc column is absorbed by the channel wall solely due to thermal conductivity, then the energy balance equation will have the form [2]

$$
\begin{equation*}
\sigma E^{2}+\frac{1}{r} \frac{d}{d r}\left(r \lambda \frac{d T}{d r}\right)=0 . \tag{1}
\end{equation*}
$$

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[^0]

Fig. 1


Fig. 2

Fig. 1. Conductivity $\sigma, 10^{2}(\Omega \cdot \mathrm{~m})^{-1}$, thermal conductivity $\lambda, 10^{-3} \mathrm{~kW} /(\mathrm{m} \cdot$ deg), and thermal conductivity function $\mathrm{S}, \mathrm{kW} /$ m , versus temperature $\mathrm{T}, 10^{3}{ }^{\circ} \mathrm{K}$, for argon plasma at $\mathrm{p}=10^{5}$ $\mathrm{N} / \mathrm{m}^{2}[3,4]$.

Fig. 2. Elliptical approximation of the function $\sigma(S)$ for argon at $\mathrm{p}=10^{5} \mathrm{~N} / \mathrm{m}^{2}, \sigma=10^{3}(\Omega \cdot \mathrm{~m})^{-1}$, and $\mathrm{S}=10^{3} \mathrm{~W} / \mathrm{m}$.

In view of the nonlinear dependence of $\sigma$ and $\lambda$ on $T$ (Fig. 1), solution of Eq. (1) is difficult. Introduction of the thermal conductivity function

$$
S=\int_{0}^{T} \lambda(T) d T
$$

transforms the original equation to the form

$$
\begin{equation*}
\sigma(S) E^{2}+\frac{1}{r} \frac{d}{d r}\left(r \frac{d S}{d r}\right)=0 \tag{2}
\end{equation*}
$$

The difference between various existing methods for solution of Eq. (2) lies in the method used for approximating the function $\sigma(\mathrm{S})$.

In Steenbeck's channe1 model [5] it is assumed that the arc consists of two regions: a conductive region ( $\sigma>0$ ) and a nonconductive one ( $\sigma=0$ ). In the conductive region the temperature and electrical conductivity are constant. In Maecker's linearization method [6], $\sigma(S)$ is approximated in the conductive region by an inclined straight line, i.e., a certain dependence of temperature on radius is considered. In Zarudi's quasichannel model [7], in contrast to the channel model, it is assumed that the temperature is dependent on radius, while the electrical conductivity, as in the channel model, is constant.

The above methods permit a quite rapid approximate calculation of the thermal and electrical characteristics of electric arcs.

A much better approximation to the exact solution is given by the linear-piecewise method of [8] and the step approximation of [7]. The first of these is based on substituting a discontinuous line for the function $\sigma(S)$. Its application requires solution of a system of transcendental equations. From a practical viewpoint the step approximation is more convenient, with calculations being performed by recurrent formulas.

It is characteristic of all these methods that one must establish beforehand an upper limit to the approximation interval. In the quasichannel model, for example, the mean electrical conductivity is calculated from the formula [7]

$$
\begin{equation*}
\sigma_{\mathrm{m}}=\int_{0}^{\mathrm{s} a} \sigma(S) d S /\left(\mathrm{S}_{a}-\mathrm{S}_{\mathrm{b}}\right), \tag{3}
\end{equation*}
$$

where $S_{a}$ and $S_{b}$ are the values of the thermal conductivity function on the axis and at the boundary of the conductive channel. With change in burning conditions the procedure for approximating $\sigma(S)$ must be repeated.

To simplify the calculation of arc characteristics the authors have employed an elliptical approximation of the function $\sigma(S)$, which is performed a single time for a given type


Fig. 3. Auxiliary function $f_{0}\left(u_{0}\right)$.
of gas at given pressure. The approximation parameters are the semiaxes $A$ and $B$ of the ellipse and the coordinates of its center $S_{C}$ and $\sigma_{C}$. Let the semiaxis $B$ coincide with the abscissa; then $\sigma_{c}=0$ and the approximating formula is written in the form

$$
\begin{equation*}
\sigma=A \sqrt{1-\left(\frac{S_{\mathrm{c}}-S}{B}\right)^{2}} \tag{4}
\end{equation*}
$$

We can determine $A, B$, and $S_{c}$ if values of the thermal conductivity function $S_{1}, S_{2}$, and $S_{3}$ corresponding to fixed values of electrical conductivity $\sigma_{1}, \sigma_{2}$, and $\sigma_{3}$ are known (Fig. 2). We choose the value $\sigma_{2}$ by the condition $2 \sigma_{2}^{2}=\sigma_{1}^{2}+\sigma_{3}^{2}$. Solving the system of equations

$$
\begin{equation*}
\sigma_{i}=A \sqrt{1-\left(\frac{S_{\mathrm{c}}-S_{i}}{B}\right)^{2}}, i=1,2,3 \tag{5}
\end{equation*}
$$

we obtain formulas for calculating $S_{C}, A$, and $B$ :

$$
\begin{gather*}
S_{\mathrm{c}}=\frac{S_{1}^{2}+S_{3}^{2}-2 S_{2}^{2}}{2\left(S_{1}+S_{3}-2 S_{2}\right)}  \tag{6}\\
A=\sqrt{\frac{\sigma_{3}^{2}\left(S_{\mathrm{c}}-S_{1}\right)^{2}-\sigma_{1}\left(S_{\mathrm{c}}-S_{3}\right)^{2}}{\left(S_{\mathrm{c}}-S_{1}\right)^{2}-\left(S_{\mathrm{c}}-S_{3}\right)^{2}}}  \tag{7}\\
B=A-\frac{S_{\mathrm{c}}-S_{3}}{\sqrt{A^{2}-\sigma_{3}^{2}}} \tag{8}
\end{gather*}
$$

The $S_{1}$ should be chosen from the condition $S_{1} \approx 0.1 S_{3}$. In this case, $\sigma_{1}^{2}\left(S_{c}-S_{3}\right)^{2} \ll \sigma_{3}^{2}\left(S_{c}-\right.$ $\left.S_{1}\right)^{2}$ and Eq. (7) takes on the form

$$
\begin{equation*}
A=\frac{\sigma_{3}\left(S_{\mathrm{c}}-S_{1}\right)}{\sqrt{\left(S_{\mathrm{c}}-S_{1}\right)^{2}-\left(S_{\mathrm{c}}-S_{3}\right)^{2}}} \tag{9}
\end{equation*}
$$

Substituting Eq. (4) in Eq. (2), we obtain the energy balance equation in the conductive region:

$$
\begin{equation*}
u^{\prime \prime}+\frac{1}{r} u^{\prime}-\frac{A E^{2}}{B} \sqrt{1-u^{2}}=0 \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
u=u(S)=\frac{S_{\mathrm{c}}-S}{B} \tag{11}
\end{equation*}
$$

and differentiation is performed with respect to $r$.
Averaging the function $f=\sqrt{1-u^{2}}$ over the interval $\left[u_{o}=u\left(S_{a}\right), u_{b}=u\left(S_{b}\right)=1\right]$, we have

$$
\begin{equation*}
\dot{t}_{0}=\frac{1}{1-u_{0}} \int_{u_{0}}^{1} \sqrt{1-u^{2}} d u=\frac{1}{2\left(1-u_{0}\right)} \quad\left(\frac{\pi}{2}-\arcsin u_{0}-u_{0} \sqrt{1-u_{0}^{2}}\right) \tag{12}
\end{equation*}
$$

where $f_{0}$ is the desired mean value. The function $f_{0}\left(u_{0}\right)$ is presented in Fig. 3.
With consideration of Eqs. (11) and (12), energy balance in the arc is described by the following equations

$$
\begin{equation*}
u^{\prime \prime}+\frac{1}{r} u^{\prime}-\frac{A E^{2}}{B} f_{0}=0 \text { for } \sigma>0, u^{\prime \prime}+\frac{1}{r} u^{\prime}=0 \text { for } \sigma=0 \tag{13}
\end{equation*}
$$



Fig. 4. Radial temperature distribution $\mathrm{T}, 10^{3}{ }^{\circ} \mathrm{K}$ : solid curves) calculation by proposed method; dashed curves) step approximation method;

1) $I=30 \mathrm{~A}$; 2) 50 ; 3) 70 ;
2) 90 ; 5) 110 .

Setting boundary conditions

$$
S(0)=S_{a}, S\left(r_{\mathrm{b}}\right)=S_{\mathrm{b}}=S_{\mathrm{c}}-B, S(R)=0
$$

from Eq. (13) we find

$$
\begin{align*}
& S=S_{a}-0.25 A E^{2} f_{0} r^{2} \text { for } r \leqslant r_{\mathrm{b}}  \tag{14}\\
& S=S_{\mathrm{b}} \frac{\ln r / R}{\ln r_{\mathrm{b}} / R} \text { for } r_{\mathrm{b}} \leqslant r \leqslant R \tag{15}
\end{align*}
$$

where the radius of the conductive region is calculated from the formula

$$
\begin{equation*}
r_{\mathrm{b}}=R \exp \left[\frac{-S_{\mathrm{b}}}{2\left(S_{\mathrm{b}}-S_{a}\right)}\right] \tag{16}
\end{equation*}
$$

which is obtained from Eqs. (14) and (15) with consideration of the continuity of thermal flux on the interregion boundary.

The formula for calculation of arc electric field intensity

$$
\begin{equation*}
E=\frac{2}{r_{\mathrm{b}}} \sqrt{\frac{\overline{S_{a}-S_{\mathrm{b}}}}{A f_{0}}} \tag{17}
\end{equation*}
$$

follows from Eq. (14) if we take $r=r_{b}$.
It also follows from Eq. (14) that

$$
\begin{equation*}
I=\frac{4 \pi\left(S_{a}-S_{b}\right)}{E}, \tag{18}
\end{equation*}
$$

inasmuch as

$$
\left.\frac{d S}{d r}\right|_{r=r \mathrm{~b}}=-\frac{I E}{2 \pi r_{\mathrm{b}}}
$$

If $S_{a}$ is unknown, but the intensity $E$ and current $I$ are given, then the axial value of the thermal conductivity function is calculated, according to Eq. (18), from the formula

$$
\begin{equation*}
S_{a}=S_{\mathrm{b}}+\frac{E I}{4 \pi} \tag{19}
\end{equation*}
$$

In form, Eqs. (14)-(18) coincide with those of the quasichannel model [7]. They differ in that, instead of $\sigma_{m}$, there appears in Eqs. (14)-(18) the quantity Afo. Upon comparison of Eq. (3) with (4), (11), and (12) it may be seen that Afor for ever $S_{a}$ is equal to $\sigma_{m}$ to an accuracy determined by the approximation of $\sigma(S)$ by the elliptical arc - Eq. (4).

The method described was used to calculate temperature distribution over the channel section of the plasmotron described above for $d=6 \mathrm{~mm}$ and number of sections $\mathrm{n}_{\mathrm{s}}=6$. Electric
field intensities of $E=636,673,730,795$, and $860 \mathrm{~V} / \mathrm{m}$ were found from the experimentally determined volt-ampere curve [1] for fixed current values of $\mathrm{I}=30,50,70$, 90 , and 110 A .

The results of calculation of the temperature distribution $T(r)$, presented in Fig. 4, are close to those of a numerical solution by the step approximation method. The following values of approximation parameters were used: $A=6342(\Omega \cdot m)^{-1}, B=7797 \mathrm{~W} / \mathrm{m}$, and $\mathrm{S}_{\mathrm{C}}=8292$ $\mathrm{W} / \mathrm{m}$. These values ensure a good approximation of the function $\sigma(S)$ for $S$ values not exceeding $8000 \mathrm{~W} / \mathrm{m}$, which corresponds to a temperature of $\mathrm{T} \simeq 15,000^{\circ} \mathrm{K}$.

In conclusion, it should be noted that the proposed method, being an analog of the coarse linearization method (quasichannel model), has the advantage that calculation of auxiliary parameters is significantly simplified.

## NOTATION

T, temperature; $\sigma$, electrical conductivity; $r$, radial coordinate; $d=2 R$, diameter of discharge channel; $\lambda$, thermal conductivity; $S$, thermal conductivity function; p, pressure; E, electric field intensity; $I$, current; $A, B, S_{C}$, parameters of approximation ellipse; $G$, gas expenditure; $u$, auxiliary variable; f, auxiliary function. Indices: b, boundary; $a$, axial.

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HEAT AND MOISTURE TRANSFER BETWEEN A FRESHLY EXPOSED
ROCK MASS AND A VENTILATING AIR JET
0. A. Kremnev, V. Ya. Zhuravlenko,

UDC 536.24:539.217.2
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The problem of heat and moisture transfer between an infinite isotropic rock mass and a ventilating air jet of constant temperature is considered. Equations are derived for the temperature- and moisture-transfer potential fields. Nomograms are presented for calculating the heat and moisture flows.

Coal mines are now sunk to depths of $1000-1100 \mathrm{~m}$. In view of the current increase in the mining of coking coals the working depth is likely to increase still further. In order to create normal labor conditions in deep shafts it is essential to introduce a system of air-cooling and to improve methods of calculating the thermal characteristics of mines.

Existing methods of calculation $[1,2]$ are based on solving the problem of transient heat transfer between the ventilating jet and the rock mass surrounding the working.

[^1][^2]
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